but the extrema of r_1 vary, depending upon which of the "paths" a,b is being considered. Thus, by inspection of Fig. 15(b), we find that

$$r_2-R\leq r_1\leq r_{b1} \quad \text{for } r_2>R, \quad \text{or path (a);} \quad (A10a)$$

$$R-r_2\leq r_1\leq R+r_2 \quad \text{for } r_2<r_b-R, \quad \text{or path (b).} \quad (A10b)$$

Upon a little reflection, and noting that all the differences appearing in the left-hand members of the inequalities (A7) to (A10) are non-negative, one further finds that (A7) and (A9) may be combined by writing

$$\max(R-r_{b1},0)\leq r_2\leq r_{b2}, \quad (A11)$$

and similarly, that (A8) and (A10) may be combined into

$$|R-r_2|\leq r_1\leq \min(r_{b1},R+r_2), \quad (A12)$$

where $\min(x_1,x_2)$ means "the smaller of the two arguments x_1 and x_2," and similarly, $\max(x_1,x_2)$ selects the larger member of the pair. Relations (A11) and (A12) give the limits shown in (A3).

Even for but a single value of the internuclear separation parameter R, however, a hand calculation of the simplified form (A3) for $\bar{\Lambda}$ still involves a prohibitive amount of labor. For this reason, the ultimate evaluation of $\bar{\Lambda}(Z,R)$ for appropriate values of Z and at separations R ranging from $0.01a_0$ to $8.0a_0$, was performed on a high-speed electronic computer.

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Polarization of 9.4-MeV Protons Elastically Scattered from Copper*

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The polarization of 9.4-MeV protons elastically scattered from copper has been measured at 5° intervals from 20° to 150° (c.m.) using a helium polarimeter with counter telescopes. In this experiment an accuracy comparable to the accuracy of some of the best differential cross-section measurements has been obtained. Optical model calculations have been made for the polarization distribution, starting with the potential used by Easlea to fit the proton differential and reaction cross-section data. By varying the strength of the spin-orbit potential it was possible to obtain a good fit to the polarization distribution. The real part of the spin orbit potential was found to be (6±1) MeV and the imaginary part of the spin-orbit potential was less than 1 MeV.

INTRODUCTION

In recent years there have been many studies of proton elastic scattering at intermediate bombarding energies. Analyses of the experimental results have shown that an optical-model potential representing the interaction between the incoming proton and the target nucleus can be used to predict the general features of the scattering data, provided that the bombarding energy is sufficiently high that compound nucleus scattering can be neglected. The parameters of this optical model potential can be found if complete elastic scattering data is available including measurements of differential cross section, total reaction cross section and polarization.

The purpose of the present experiment was to investigate in detail a scattering process in which the nuclear scattering is expected to be predominantly shape elastic, and, therefore, describable by an optical model potential. Copper was chosen for this investigation because: The compound-elastic scattering is small, the (p,n) thresholds being well below the incident proton energy; the copper nucleus is not deformed so that one is justified in using spherical potentials in analyzing the results; the Coulomb barrier is sufficiently low that the

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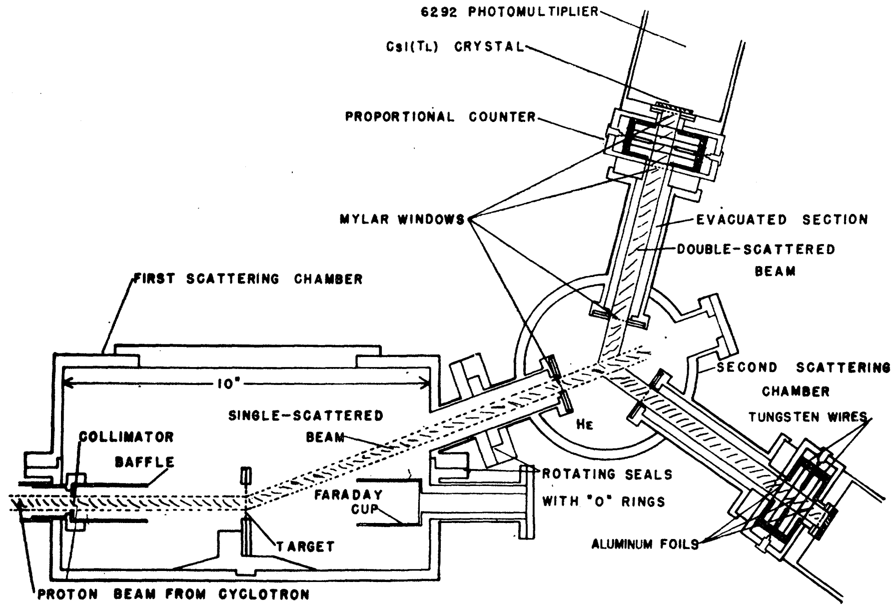

FIG. 1. Schematic drawing of polarization apparatus showing the polarimeter adjusted so that the first and second scattering planes coincide for a first-scattering angle of 20°.

nuclear scattering effect can be observed in the angular distribution except at the smallest scattering angles. Hintz,[1] and Greenless and Jarvis[2] have made accurate measurements of the differential scattering cross section for protons on copper at 9.75 and 9.47 MeV, respectively, and Benveniste, Booth, and Mitchell[3] have made similar measurements for Cu^{63} and Cu^{65} at 8.8 and 10.2 MeV. Total reaction cross sections for protons on copper have been measured directly by Greenless and Jarvis[2] at 9.3 MeV and indirectly by Meyer and Hintz[4] and Albert and Hansen[5] at 9.85 MeV. Polarization measurements for protons elastically scattered from copper at about 10 MeV have been made by Rosen, Brolley, and Stewart[6] using photographic emulsions. In the measurements described here a polarization distribution has been obtained with an accuracy comparable to the accuracy of some of the best differential cross-section measurements.

Optical-model analyses of the proton elastic scattering data near 10 MeV have been discussed by Nodvick and Saxon[7] and by Easlea.[8] Nodvick and Saxon found that almost equally good fits could be obtained over a range of values of the radius constant, R, but their predicted values for the reaction cross sections were about 200 mb lower than the experimental value subsequently found. Easlea[8] found that a surface-absorption potential rather than a volume-absorption potential was required in order to fit the observed value of 930 mb for the total reaction cross section. The previous experimental polarization data were not precise enough to determine accurately the spin-orbit potentials but were consistent with the optical model predictions.

METHOD

Bombardments were made with the external beam of the University of Birmingham 60-in. cyclotron, using 9.6-MeV protons. H^+ ions were extracted from the cyclotron through a steel tube which provided magnetic shielding for the protons. The beam was brought through a quadrupole focusing magnet, a $\frac{3}{8}$-in.-diam tungsten collimator, and onto a copper target, with a maximum intensity of 25 μA.

The polarization measurements were made by means of a double-scattering technique in which a copper foil was used for the first target and helium gas was used for the second target. The apparatus used for these polarization measurements consisted of two scattering chambers as shown in Fig. 1. The first scattering chamber was made in two halves, the proton beam entering the lower half, passing through the copper target and into a Faraday cup. The upper half of the first scattering chamber rotated about the lower half and carried the second scattering chamber and detectors. The position of the upper half of the first scattering chamber relative to the lower half determined the angle of the first scattering. This arrangement enabled measurements to be made in the angular range 20°–160° for the first scattering.

The second scattering chamber and detectors, forming a polarimeter, could be rotated about an axis in the direction of the proton beam after the first scattering so

[1] N. M. Hintz, Phys. Rev. **106**, 1201 (1957).
[2] G. W. Greenless and O. N. Jarvis, Proc. Phys. Soc. (London) **A78**, 1275 (1961).
[3] J. Benveniste, R. Booth, and A. Mitchell, Phys. Rev. **123**, 1818 (1961).
[4] V. Meyer and N. M. Hintz, Phys. Rev. Letters **5**, 207 (1960).
[5] R. D. Albert and L. F. Hansen, Phys. Rev. **123**, 1749 (1961).
[6] L. Rosen, J. E. Brolley, Jr., and L. Stewart, Phys. Rev. **121**, 1423 (1961).
[7] J. S. Nodvik and D. S. Saxon, Phys. Rev. **117**, 1539 (1960).
[8] B. R. Easlea, Proc. Phys. Soc. (London) **A78**, 1285 (1961).

that the planes of the first and second scatterings would coincide. Figure 1 shows the polarimeter oriented for a polarization measurement in which the angle for the first scattering is 20° and both planes are vertical. For each angle of observation in the first scattering the polarimeter was rotated to make the two scattering planes coincide.

The polarimeter contained helium gas at a pressure of 10 atm, with windows of 0.002-in.-thick Mylar foil. The angles for the second scatterings were 55° to the left and right of the direction of the first-scattered beam. The polarization was measured by detecting simultaneously the number of protons scattered to the left and to the right. The maximum angular spread for the second scattering was ±5° and for the first scattering was ±2.5°. The mean angular spreads were approximately half these values.

The double-scattered protons were detected with counter telescopes, each consisting of a proportional counter and a scintillation counter. A multiwire proportional counter was used as a flow-counter with an argon (90%)-methane (10%) mixture which produced an energy loss of 200 keV for protons passing through it. The active volume of the counter was made small in order to reduce background. The scintillation crystal was a 1-in.-diam Cs(Tl) crystal with a thickness slightly greater than the range of the doubly scattered protons. The scintillation crystals were coupled to Dumont 6292 photomultiplier tubes in a conventional way.

Coincidence pulses from the counter telescopes were used to gate a 100-channel pulse-height analyzer on which were recorded the pulses from the scintillation detectors. A pedestal pulse was added to the pulses from one of the detectors so that the pulses from the left and right telescopes could be recorded simultaneously in the pulse-height analyzer. The widths at half-maximum of the elastically scattered proton peaks were about 500 keV, the inelastic scattering and background being sufficiently small that the number of counts in the elastic peaks could be obtained with a small correction. A typical pulse-height spectrum is shown in Fig. 2. The

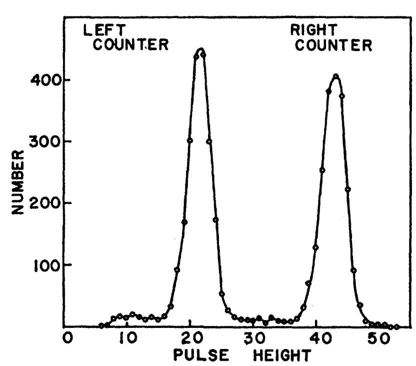

FIG. 2. Pulse-height distribution of protons scattered into the left and right counter telescopes from copper at θ(c.m.) = 45°.

polarization was calculated from the asymmetry measurements as follows:

$$\epsilon = (N_L - N_R)/(N_L + N_R) = P_1(\theta_1)\langle P_2(\theta_2)\cos\phi_2\rangle_{\text{av}}, \quad (1)$$

where N_L and N_R were the number of elastically scattered protons detected in the left and right counter telescopes. The value of $\langle P_2(\theta_2)\cos\phi_2\rangle_{\text{av}}$ was -0.613 at 8 MeV as calculated from Brockman's helium polarization results[9] and the geometry of the polarimeter by numerical integration:

$$\langle P_2(\theta_2)\cos\phi_2\rangle_{\text{av}} = \frac{\int \sigma_2(\theta_2)P_2(\theta_2)\cos\phi_2 d\Omega_2}{\int \sigma_2(\theta_2)d\Omega_2}. \quad (2)$$

The error in the helium polarization value should not limit the accuracy of the present results.

Instrumental asymmetries of 1 to 2% which arose in the mechanical construction of the polarimeter, in the position of the beam on the target, in the inhomogeneity of the target and in the electronics were corrected by making a series of four asymmetry measurements. Sets of asymmetry measurements were made for identical left and right first-scattering angles; in each case the counter telescopes were set in the plane of the first scattering for a first asymmetry measurement. With this procedure the various instrumental asymmetries were obtained for each measurement and corrected for.

EXPERIMENTAL RESULTS

The polarization measurements were made using a natural copper target of spectroscopic purity with a thickness equivalent to 390 keV for the incident proton beam. Heating effects on the target limited the beam intensity to 25 μA. The energy of the incident proton beam was 9.43 MeV at the center of the target, and the energies of the scattered protons were between 8.0 and 8.5 MeV at the center of the helium polarimeter and between 5.0 and 5.5 MeV in the scintillation crystal, depending on the angle of the first scattering. The angle of the second scattering was chosen so as to provide a high value for $P_2(\theta_2)$ which was constant with energy in the range 8–8.5 MeV.

An angular range of 20°–150° (c.m.) for the first scattering was covered in 5° intervals. At least 10 000 counts were recorded at each angle, making it possible to obtain asymmetry results at angles forward of 90° with an accuracy of better than 1%. At the backward scattering angles the error was somewhat larger because of background subtraction. The copper polarization results were calculated using Eq. (1) and are given in Table I, using the Basel sign convention.[10] The errors which are quoted are the probable errors arising from

[9] K. W. Brockman, Phys. Rev. **110**, 163 (1958).

[10] *Proceedings of the International Symposium on Polarization Phenomena of Nucleons, Basel, 1960* [Suppl. Helv. Phys. Acta **6**, 436 (1961)].

TABLE I. Polarization results for 9.13-MeV protons elastically scattered from copper, using the Basel sign convention.

θ(c.m.)	$P(\theta)$	θ(c.m.)	$P(\theta)$
20	-0.5 ± 0.7	90	20.6 ± 2.4
25	0.7 ± 1.2	95	16.1 ± 2.5
30	-0.3 ± 1.2	100	10.4 ± 2.7
35	0.8 ± 1.2	105	5.3 ± 2.9
40	-0.3 ± 1.1	110	-5.0 ± 2.9
45	-5.0 ± 1.2	115	-12.8 ± 3.8
50	-7.2 ± 1.5	120	-15.8 ± 3.7
55	-10.5 ± 1.9	125	-24.5 ± 4.1
60	-12.3 ± 2.1	130	-37.3 ± 5.0
65	-9.9 ± 1.5	135	-5.2 ± 4.6
70	5.8 ± 1.4	140	16.7 ± 3.9
75	21.2 ± 2.5	145	51.6 ± 8.7
80	29.0 ± 2.7	150	59.7 ± 6.7
85	23.6 ± 2.6		

statistics and our best estimate of the systematic errors present in the experiment.

The present polarization results show greater maximum and minimum values than found in previous measurements at 10 MeV,[6] presumably due to the better angular definition used in the present experiment. Since each of the present measurements covers only a small range of angles compared with the period of oscillation of the diffraction pattern, the results should give realistic values for the polarization maxima and minima.

DISCUSSION

Optical-model calculations have been made, using a program developed for the Mercury Computer by Easlea,[8] who kindly made this program available to us. His analysis was used as a starting point for the present work in which the effect of the spin-orbit potentials was studied. Easlea showed that the absorption must be peaked at the surface of the nucleus in order to fit both the differential elastic scattering cross section and the total reaction cross section. The potential used in the present investigation included both volume- and surface-absorptive potentials and both real and imaginary spin-orbit potentials. This potential can be written in the following form:

$$V(r)=V_c(r)+Vf(r)+iWg(r)+(v+iw)h(r)\boldsymbol{\sigma}\cdot\mathbf{l}, \quad (3)$$

where $V_c(r)$ is the Coulomb potential. The radial dependence of the nuclear potential is given by:

$$f(r)=\{1+\exp[(r-R)/a]\}^{-1}, \quad (4)$$

$$g(r)=Af(r)+B(df/dr), \quad (5)$$

$$h(r)=a_0^2(1/r)(df/dr), \quad (6)$$

where A and B are constants, and $R=r_0A^{1/3}$. The imaginary part of the central potential is specified by quoting W_c, the value of $Wg(r)$ at $r=0$, and W_s, the value of $Wg(r)$ at $r=R$. Unfortunately, the spin-orbit potentials have been written in several forms by different authors. The form used here is that of Nodvik and Saxon[7] in which the constant $a_0^2=2.0$ F.[2] As the

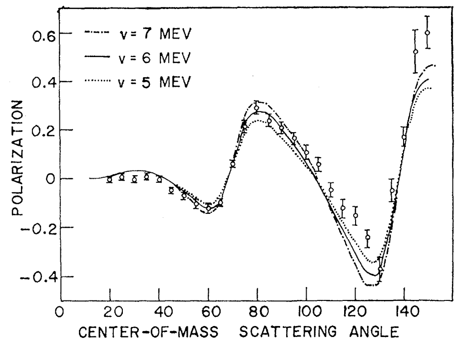

FIG. 3. Polarization distribution for 9.43-MeV protons elastically scattered from copper. The curves are the optical model calculations with the following parameters: $V=-47$ MeV, $W_s=-8.2$ MeV, $W_c=0$, $v=5.0, 6.0, 7.0$ MeV, $w=0$, $r_0=1.30$ F, $a=0.65$ F.

potential is written here, a negative value for V corresponds to an attractive potential, a negative value for W corresponds to an absorptive potential, and a positive value for v corresponds to an attractive potential for protons with $J=l+\frac{1}{2}$ and a repulsive potential for protons with $J=l-\frac{1}{2}$. A nonzero value for w would account for a difference in the density of $J=l+\frac{1}{2}$ and $J=l-\frac{1}{2}$ levels.

In order to test the applicability of the optical model potential in describing the proton scattering from Cu we started with Easlea's best fit[8] to the 9.47 MeV differential cross-section data and 9.3 MeV total reaction cross-section data.[2] This potential which was found by Easlea before the present polarization experiments were begun, fit these polarization results nearly as well as it did the differential cross section results. Optical-model calculations with different sets of parameters produced no essential improvement in the fits to the experimental data, although an exhaustive parameter search was not attempted. The best optical-model fit to the polarization data with different values of the real spin-orbit potential v is shown in Fig. 3. An optical-model fit with different values of the imaginary spin-orbit potential w is shown in Fig. 4. The fit to Greenlees

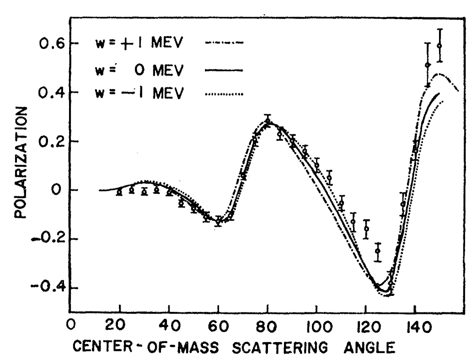

FIG. 4. Polarization distribution for 9.43-MeV protons elastically scattered from copper. The curves are the optical model calculations with the following parameters: $V=-47$ MeV, $W_s=-8.2$ MeV, $W_c=0$, $v=6.0$ MeV, $w=-1, 0, +1$ MeV, $r_0=1.30$ F, $a=0.65$ F.

and Jarvis differential cross-section results using the optical-model potential which gave the best fit to the polarization results is shown in Fig. 5. This potential predicts a value of 920 mb for the total reaction cross section, which compares well with the experimental value of 930±70 mb found by Greenlees and Jarvis. The optical-model calculation of the polarization was

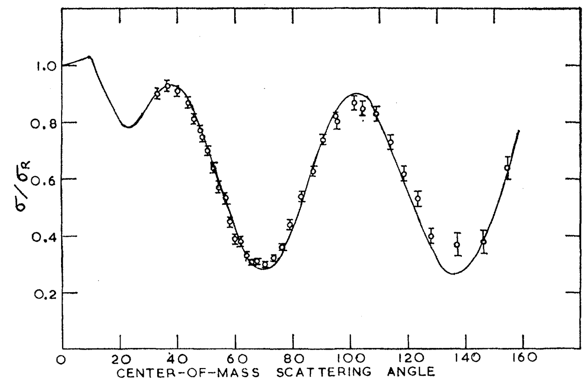

FIG. 5. Differential cross-section data for 9.47-MeV protons elastically scattered from copper (reference 2). The smooth curve is the optical-model calculation with the following parameters: $V=-47$ MeV, $W_s=-8.2$ MeV, $W_c=0$, $v=6.0$ MeV, $w=0$, $r_0=1.30$ F, $a=0.65$ F.

not very sensitive to the ratio of surface to volume absorption as long as W_s/W_c is greater than about 2/1. As a result of varying the spin-orbit potential we can assign the following value for 9.4-MeV protons on copper:

$$v=(6\pm1)\text{ MeV}. \tag{7}$$

The effect of the imaginary part of the spin-orbit potential on the polarization distribution was also investigated and the absolute value of w was found to be less than 1 MeV.

Nodvik and Saxon found a value of $v=4$ MeV as a result of a systematic parameter search in fitting the Hintz 9.75-MeV cross-section data and the Rosen 10-MeV polarization data. An optical-model analysis of proton-nucleus elastic scattering data by Bjorklund, Campbell, and Fernbach[11] indicated that $v=(5\pm1)$ MeV for proton energies below 14 MeV. Data for protons scattered from several different elements at 8 MeV and 10 MeV have been fit by Rosen *et al.*[6,12] with an optical-model potential in which the spin-orbit potential is equivalent to a value of $v=7$ MeV.

The values of v found by Bjorklund *et al.* and Rosen *et al.* agree rather well with our evaluation. The value of $v=4$ MeV found by Nodvik and Saxon seems to be too small. Nodvik and Saxon used the optical potential with volume absorption, which didn't fit the experimental value of the total reaction cross section, and tried to fit Rosen's polarization data for copper which had somewhat smaller maximum and minimum polarization values than were found in the present experiment.

The spin-orbit potential required for the optical-model description of proton scattering from copper at 10 MeV is 23 times the Thomas term, while the spin-orbit potential usually used in shell model calculations (where $E_p=-8$ MeV) is 40 times the Thomas term.[13] The comparison of the spin-orbit potentials is not unambiguous since in addition to the proton energies being different, the central potentials which have been used in the analyses are not the same in the two cases; most particularly, the absorptive potential used in the shell model calculations must be zero.

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[11] F. Bjorklund, G. Campbell, and S. Fernbach, in *Proceedings International Symposium on Polarization Phenomena of Nucleons, Basel, 1960* [Suppl. Helv. Phys. Acta **6**, 432 (1961)].
[12] L. Rosen, J. E. Brolley, Jr., M. L. Gursky, and L. Stewart, Phys. Rev. **124**, 199 (1961).
[13] A. A. Ross, H. Mark, and R. D. Lawson Phys. Rev. **102** 1613 (1956).